A PROGRAMMED SEQUENCE ON

EXPONENTIAL NOTATION

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CHEMICAL EDUCATION MATERIAL STUDY

W. H. FREEMAN AND COMPANY Cooperating Publishers SAN FRANCISCO AND LONDON



EXPONENTIAL NOTATION

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Work in science often requires very large or very small numbers such as 602,300,000,000,000,000,000,000. A convenient way to handle these is through exponential notation, which this booklet explains. There are five parts

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How to Use This Booklet

This coverage of exponential notation is designed to be undertaken by you on a self-instructional basis. Using a small sheet of paper or cardboard as a mask, you are to conceal the answers which are listed in the right hand columns of the booklet until you have actually written down your own response on a separate sheet of paper. Do not mark this booklet itself so others will be able to use it again.

This trial material was developed by CHEM Study to facilitate computational work in the high school chemistry course.

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Printed in the United States of America.

POWERS OF TEN AND EXPONENTIAL NOTATION

In science there is often a need to work with very large and very small numbers. From the distances of space to the distances in molecules, we are concerned with measurement.

This booklet is designed to show you a way of expressing and using very large and very small numbers. It is intended that this should be undertaken as self-instruction on your part should you need either a rapid review or an introduction to this topic. Using a small sheet of paper or cardboard as a mask, conceal the answers in the right hand margin until you have actually written down your own response on a separate sheet of paper. DO NOT MARK THIS BOOKLET.

	Let's start with some moderate sized numbers as examples. Just as 20 may be written as 2 x 10,	
1	so may 40 be written as,	4 x 10
2	so, too, 70 may be written as	7 x 10
	We can also write 300 as 3 x 100	
3	and 500 as	5 x 100
4	and 800 as	8 x 100
	We can go even further; into the thousands.	
5	6000 = 6 x	1000
6	7000 = x 1000	7
7	or 9000 = x	9 x 1000
8	80,000 = x 10,000	8
9	Since 100 = 10 x 10, we see that we can factor 100 into 2 equal parts. Likewise, since 1000 = 10 x 10 x 10, we can factor 1000 into equal parts. We can more conveniently indicate these equal factors using exponents. For example,	3
.0	$100 = 10 \times 10 = 10^{2}$ And, $1000 = 10 \times 10 \times 10 = 10^{?}$	103
.1	So, in place of the 1000 in (6 x 1000) we could write $6 \times 10^{?}$	10 ³

```
Note that we now have 6000 = 6 \times 10^3. In a similar
      way, since 50,000 = 5 \times 10,000, we could write
                      50,000 = 5 \times 10^4
12
     Note that there are zeros in 50,000, and that
                                                                 4
      the exponent of 10 is also ____.
13
                                                                 17
      In 20.000,000 = 2 \times 10^7.
14
      the exponent is .
                                                                 7
     The number of zeros in 20,000,000 is
15
                                                                 7
      The relationship here seems to be that the value
                  of 10 is the same as the number
16
                                                                 exponent
17
                in the original numeral.
                                                                 zeros
      Using this relationship, complete the following:
                                                                 106
                       3.000.000 = 3 \times 10^{?}
18
      Change the following numerals to this exponential
      form:
                                                                 3 \times 10^{2}
           a) 300
19
                                                                 8 \times 10^{4}
20
           b) 80,000
                                                                 1 \times 10^{5}
           c)
              100,000
21
      Let's drop back for a moment to some smaller numbers.
      If we can write
                      25 as 2.5 x 10.
                  and 39 as 3.9 \times 10,
           we can also write 45 as ____ x 10.
                                                                 4.5
22
      We can also write 87 as ____ x 10.
23
                                                                 8.7
      When we jump up into the hundreds we can use the same
      system.
                       350 can be written as 3.5 \times 100
24
                  and 630 can be written as 6.3 x
                                                                 100
     But, we know 100 can be written as 10^2
25
          so we now write 6.3 \times 100 as \times 10^2.
                                                                6.3
     In a similar way, 5,600 can be written as
                     5.6 x 1000
              or as = x 10^3.
26
                                                                5.6
```

Let's look again at the last expression in the preceding box.

```
We have 5,600 = 5.6 \times 10^3.
     In changing 5,600 to 5.6, we moved the decimal
     point
                  places.
     If your last answer in the above box was correct, go
     on to the next box.
     If you're puzzled as to where the decimal point is in
     the numeral 2758, continue on here.
28
     When a numeral such as 2758 is written without a
     decimal point, where do we assume the decimal point
     to be?
                           In front of the 2.
                           Between the 7 and the 5.
                          After the 8
                                                                   After the 8.
     So, 2758 = 2758.
29
     Where is the decimal point understood to be in the
     numeral 142?
                                                                   142.
                                                                   (After the 2.)
     If it is not indicated, the decimal point is assumed
30
     to follow the ____ digit of a number.
                                                                   last
     In changing 5,600 to 5.6 x 10^3 the decimal point was
31
    moved
                 places.
                                                                   three
     In changing 5600 to 5.6 x 10^3 we found that the decimal point was moved 3 places. It's important to notice that the exponent in this expression
32
     is also _
                                                                   3
     As another example, take the following
                       73,400 = 7.34 \times 10^4.
      We note that in changing 73,400 to 7.34 the decimal
     was moved places, and that the exponent used
33
34
      is also _
      We might even go so far as to state this as a rule of
      thumb, namely that the decimal point change equals
                  of 10.
35
                                                                   exponent
36
      Try this rule in determining the exponent in the
      following expression:
                       3678 = 3.678 \times 10^{?}
                                                                   103
      If your answer was correct, go to the next, box.
      If you are wondering how this last answer was obtained,
      or would like to have more practice, copy down the
      following, including the letters:
                       3678 = 3.678 \times 10^{?}
                                  В
                                           C
      In the numeral 3678 (Part A), put in the decimal
37
                                                                    3678.
      point.
```

```
(If this last answer is puzzling to you, you may want to review Items 28 to 30.)
      In changing 3678. in A to 3.678 in B, how many places would you move the decimal point?
38
                                                                          3
      Since you moved the decimal point 3 places, then the
      exponent in part C would also be _____.
39
      Now complete your problem: 3678 = 3.678 \times 10^{?}
                                                                         103
40
                                                                         10<sup>2</sup>
                                                       7.89 \times 10^{?}
      Copy and complete the following: 789 =
41
      If your answer was correct, go to the next box.
      If you are not sure just how this result was obtained,
      copy down the problem given below, including the
      letters:
                         789 = 7.89 \times 10^{?}
                                  B C
      In changing the numeral at A to the numeral at B you would move the decimal point _____ places.
                                                                          2
42
      Therefore the exponent of 10 (at C) would also
43
                                                                          2
      be _____.
                                                                          10<sup>2</sup>
      Now complete your problem: 789 = 7.89 \times 10^{?}
44
45
      Copy and complete the following:
                                                                          107
                            16,000,000 = 1.6 \times 10^{?}
      If your answer was correct, jump to the next box.
      If you want to go through this problem in more detail,
      copy down the following, including the letters:
                            16,000,000 = 1.6 \times 10^{\circ}
                                               B
      Locate the decimal point in part A.
46
                                                                          16,000,000.
      In changing the numeral at A to the numeral at B, you would move the decimal point _____ places.
47
      Therefore the exponent in part C would have a value
48
      of _____.
49
      Now complete your problem
                            16,000,000 = 1.6 \times 10^{?}
                                                                          16,000,000
                                                                          = 1.6 \times 10^7
      by inserting the proper exponent.
      Complete the following:
                                                                          484,000 =
                            484,000 = 4.84 x
                                                                          4.84 \times 10^{5}
50
      Copy and complete the following:
                                                                          363,000 =
                            363,000 = 3.63 ____
                                                                          3.63 \times 10^{5}
51
      In the last answer, note that the "x" stands for "times", and must be included.
```

52	Copy and complete: 140,000,000 = 1.40	1.48 x 10 ⁸
53	Copy and complete: 14,500 =	1.45 x 10 ⁴
	In looking back over the answers that you have written you'll notice that they are in the following general form: $2500 = 2.5 \times 10^{3}$	
	A B C	
54	Notice particularly that in the part labeled B there is always only digit to the left of the decimal point.	one
	$2500 = 2.5 \times 10^3$ A B C	
55	The "standard form" for exponential notation is represented by parts B and C in the above expression. There is only one digit to the left of the	decimal point
56	The following 3 numerals are all equal, but only one is in the standard form. Which one is it? 23.5 x 10^3 2.35 x 10^4 0.235 x 10^5 A B C	2.35 x 10 ⁴
57	We recognize this as the "standard form" because it has only digit to the left of the	one, decima point
58	Change 8500 to the standard form in exponential	8500 =
90	notation.	8.5 x 10 ³
	If you got this answer correct, go to the next box.	
	If you'd like a little fuller explanation of this last problem, copy down the following exactly as shown.	
	8500 =	
	A B C	2
59	Mark the decimal point in part A.	8500. A
60	Change the numeral of A so there is only one digit to the left of the decimal point, and copy it above B on your paper.	8.5 B
	Note that in B we've just dropped the extra zeros since they only serve to fix the decimal place in the original numeral.	
	You now have 8500. = 8.5 A B C	
61	In changing the numeral from A to B, you moved the decimal point places.	3
	Therefore, the 10 in part C will have an exponent	
62	of	3
63	Put in the 10 with this exponent in part C.	10 ³

	Now you have: 8500. = 8.5 10 ³	
64	The only thing missing from a correct answer now is the " " sign which goes in the blank space. (Put it in your answer if you haven't done so	"times"
	Now your expression is complete and looks like this:	
Į	$8500. = 8.5 \times 10^3$	
65	Change the distance from the earth to the sun, 93,000,000 miles, to the standard exponential form.	9.3 x 10 ⁷ miles.
66	One estimate places the world population at 2,870,000,000 people. Express this in exponential notation.	2.87 x 10 ⁹ people.
ſ		
66	Change the following number to exponential notation: 602,000,000,000,000,000,000	6.02 x 10 ²³
ī		
	We may want to change an exponential notation back to its usual numerical form. Suppose, for example,	
67	we want to change 2.5 x 10 ³ back to its usual numerical form. Here the exponent is	3
01	This means that the decimal point will also be	<i></i>
68	shifted by places.	three
	Thus, $2.5 \times 10^3 = 2500$	
	From this example we can see that the shift in the	exponent
69	decimal point is determined by the of the 10.	(power)
70	Change 1.49 x 10 ⁴ to its usual numerical form.	14,900
10	If you got this right, and you think you have the	14,900
	idea of this change, go to the next box. For a detailed coverage of this problem continue on here.	
71	In 1.49 x $10^4 = \frac{?}{}$, what is the exponent of 10?	1
72	Therefore, how many places will we move the decimal point?	4 places
73	When you move the decimal point 4 places in the numeral 1.49, what new numeral do you get?	14,900
	In case your last answer was 0.000149, remember we have been dealing, thus far, in numbers which are	
74	(smaller, larger) than 1.	larger
75	Now you can complete the problem, 1.49 x 10 =	14,900

76	Change 4.7×10^5 to its usual numerical form.	4.7 x 10 ⁵ = 470,000
	If you made this conversion easily, move on to the next box.	
	For a detailed coverage of this conversion, recopy the problem:	
	$4.7 \times 10^5 = ?$	
77	What is the value of the exponent in this expression?	5
	Since the exponent is 5, when the 4.7 is converted to the usual numerical form, the decimal point will be	_
78	shifted places.	5
79	Make this shift in the decimal point of 4.7.	470,000
80	Insert this as your conversion in $4.7 \times 10^5 = $	470,000
81	Change 1.84 x 10^{13} to the usual numerical form.	18,400,000,
82	Change 3 x 10 ³ from its exponential form.	3000
83	Change 9.9 x 10 ⁷ from its exponential form.	99,000,000
84	Thus far, all our work with exponential notation has been with numbers which have been (larger, smaller) than one.	larger
85	It is also quite possible to use this same exponential notation to represent numbers which are one as well as numbers which are larger than one.	smaller
		1
	Compare these two exponential forms:	
	2.5 x 10 ³ 2.5 x 10 ⁻³ B	
	A and B represent two different numbers. The only	
86	difference between them is that B has a	negative
87	exponent, while A has a positive	exponent
	We are comparing 2.5×10^3 and 2.5×10^{-3} B	
	From our previous work we recognize that A represents	
88	a relatively (large, small) number.	large
89	In contrast to A, B represents a relatively (large, small) number.	small
	We might then expect that since the positive exponent in A represents a relatively large number, the nega-	
90	tive exponent in B represents a relatively number.	small

	Thus, small numbers that is, less than one in	
91	value will have a (positive, negative) exponent in exponential notation.	negative
	All the manipulations we have been doing will still apply. We'll just have to note that numbers less	
92	than 1 will have a exponent.	negative
93	Thus a negative exponent is used in the exponential notation if the number is than one.	less
	Copy the following example, including the letters:	
	$0.0018 = 1.8 \times 10^{?}$	
	A B C	
	In going from the numeral at A to that at B, the	
94	decimal point was moved places.	3
	Since the numeral at A is less than one, the exponent	
95	in C will be (+3, 3, -3).	-3
96	Now complete the problem $0.0018 = 1.8 \times 10^{?}$	10-3
	One thing you ought to note in this example,	
	$0.0018 = 1.8 \times 10^{-3}$	
97 98	is that the numeral in B still has only digit to the left of the .	one
90		decimal point
	So it is in our standard form.	
99	Copy and complete the following: $0.036 = 3.6 \times 10^{?}$	10-2
	If you were correct and feel you have this operation down pretty well, go on to the next box.	
	For a more detailed coverage, copy down the following, including the letters:	
	$0.036 = 3.6 \times 10^{?}$	
	A B C	
	A is our original numeral. We are going to change it to the standard form at B by moving the decimal point	
100	places.	2
101	Our original numeral at A is (greater, less) than one.	less
102	This means our exponent in C will be (+, -).	- (negative)
	So, our exponent will be 2 (same as the decimal point shift) and negative (the numeral is less than one).	
103	Put this exponent in your problem.	0.036 =
		3.6 x 10 ⁻²
104	Copy and complete the following: 0.0007 = 7	0.0007 =
	The sample of the relationships of the sample of the sampl	7×10^{-4}

	If you understand this last problem, jump to the next box.	
	If you'd like a more detailed coverage, copy down this problem, including the letters:	
	0.0007 = 7 A B C	
105	A B C Right after the 7, put in the symbol for "times".	0.0007 =
	The state of the s	7 x
106	Next, insert the base 10 at C.	This gives us 0.0007 = 7 x 10
	(These two parts are not really equal, yet, but we'll leave the "equals" sign in, anyhow.)	
107	Now we need to determine the proper exponent for part C. The numeral in A is than one.	less
107	This tells us that the exponent of the 10 will be	1000
108	(positive, negative).	negative
109	In changing the numeral from A to B, the decimal point was shifted places.	4
110	So, we know that the exponent of ten will be $(+4, -4)$.	-4
111	Now, complete this problem by inserting the correct exponent.	0.0007 = 7 x 10 ⁻⁴
1		
112	Copy the following and change it to exponential notation:	
	0.00000000013 = x	0.00000000013 1.3 x 10 ⁻¹⁰
	If you were able to do this easily, hop on to the next box.	
	To go through this solution more deliberately, copy down the following, including the letters:	
	0.0000000013 = x	
113	A B C In the standard exponential form, what will the	
113	numeral at B be?	1.3
	Insert this value in your problem, and include the times sign and the base 10.	
	Now your problem looks like this.	
	$0.00000000013 = 1.3 \times 10$	
114	What is still missing from this example?	The exponent of 10.
115	Will this exponent be positive or negative?	negative (A is less than one.)

116	How many places was the decimal point moved in changing the numeral at A to that at B?	10
117	So, the exponent of the base 10 will be	-10
	(You did use the correct sign, didn't you?)	
118	If so, put this exponent in your problem to complete it.	0.00000000013
		$= 1.3 \times 10^{-10}$
119	Change the following to exponential notation:	
	0.0000178	1.78 x 10 ⁻⁵
	If you've got the idea, jump ahead to the next box.	
	For a closer look at how we got this answer, copy down the following:	
	0.0000178 =	
	A B C	
120	Insert the proper numeral at B and put in the times sign.	0.0000178 = 1.78 x
121	What base will you put at C?	10
	Put the base 10 at C.	
	This gives you the following:	
	$0.0000178 = 1.78 \times 10$	1
	A B C	
122	What is still missing from this expression?	The exponent of 10.
123	What should the exponent be?	- 5
	If you see where this last answer came from, insert it in your problem and go to Item No. 128.	
	To see where this answer came from, count the number of places the decimal point was shifted in changing the numeral at A to the numeral at B. If your answer sheet is not clear, look back to Item No.121. The number of places the decimal point was shifted	
124	was	5
.125	Is A greater than one or less than one?	less than one
126	Therefore the exponent will be (positive, negative).	negative
127	Thus, our exponent will be	- 5
128	Insert this exponent in your expression to complete this problem.	0.0000178 = 1.78 x 10 ⁻⁵
129	Change 0.015 to exponential form.	1.5 x 10 ⁻²
130	Change 0.1 to exponential notation.	1 x 10 ⁻¹

	The reverse of this operation is naturally possible. In doing this, it necessary to note the sign of the exponent of 10 to determine if the numeral will be	
131	smaller than one or than one.	larger
	Suppose we were to change 2.7×10^{-7} to the usual numerical form.	
132	Will the numeral be smaller or larger than one?	smaller than one
133	How did you know this?	The exponent is negative.
134	In changing 2.7×10^{-7} to its usual numerical form, how many places will the decimal point be moved?	7 places
135	How did you know this?	From the value of the exponent.
	When the decimal point of 2.7 is moved 7 places, two different numerals could result:	
	0.00000027 or, 27,000,000	
	АВ	
136	Which of these is correct for 2.7×10^{-7} ?	0.00000027
137	We know the numeral must be ${}$ than one since than one since the exponent is negative.	less
138	Change 3.4×10^{-3} to the usual numerical form.	0.0034
	If this was O.K., go to the next box.	
	For a more detailed explanation, copy the following:	
	$3.4 \times 10^{-3} = $	
139	What is the exponent of 10?	- 3
~ 37	Therefore the decimal point of 3.4 (B) will be moved	
140		3
141	The numeral which results from this decimal point shift should be (less, greater) than one.	less
	(The exponent was negative, wasn't it?)	
142	What will be the numeral for part A when the decimal point is shifted 3 places to give a numeral less than one?	0.0034
]
143	Change 9.9 x 10 ⁻⁹ from its exponential notation.	0.0000000099

You may want to practice with the problems of Exercises 1 and 2, page 116 of your CHEM Study Laboratory Manual.

SHIFTING DECIMAL PLACES

IN EXPONENTIAL NOTATION

It is sometimes necessary to shift the decimal point in our exponential notation while performing some of the arithmetical operations with powers of ten.

We are now used to the exponential notation which is used to represent numbers. For instance, 3.45 x 10 is in the proper form for exponential notation because it has a decimal numeral with digit to the left of the decimal point and is multiplied by a power of _____.

1

Sometimes it is necessary to change this standard form of exponential notation to get either a different decimal numeral or a different power of

10

However, no matter how we change the decimal numeral and the power of ten, we must not change the value of the expression.

1500

What is the usual decimal form of 1.5×10^3 ?

In exponential notation we express this number as $1500 = 1.5 \times 10^3$.

10

If we want to change the exponential form of this number, we <u>could</u> change the decimal numeral part so long as we <u>also</u> changed the power of _____ so that the new expression would still have the <u>same</u> value.

Preliminary to doing this, though, we need to be able to compare numbers correctly.

Which represents the larger number, 2.4×10^7 or 1.5×10^7 ?

Which is larger, 10⁵ or 10⁸?

5

6

7

8

Which is larger, 1.7×10^3 or 1.7×10^4 ?

Which is larger, 6.7×10^3 or 3.4×10^4 ?

2.4 x 10⁷ is larger.
10⁸ is larger.
1.7 x 10⁴ is larger.
3.4 x 10⁴ is larger.
(Note the exponents.)

If you are in doubt as to which of two numbers shown in exponential notation is larger, you can always go back to the decimal form of the numbers and then compare them.

9	Suppose you came across a number written as 354.5 x 10 ⁶ and you wanted to change it to the standard exponential form. This means the decimal numeral part should have only digit to the left of the decimal point.	1
10	This means you will want to change 354.5 to	3.545
11	In changing 354.5 to 3.545, you moved the decimal point(how many) places.	two
12	Since you moved the decimal point two places, you will also have to change the exponent by	two
	The exponent has to be changed by 2, but should it be 2 more, or 2 less. This is what we have to decide. Our problem is to change 354.5×10^6 to $3.545 \times 10^?$.	
13	The value of the expression must not be changed. So, since we have made the decimal part of the expression smaller, we must counteract this by making the power of ten part (larger, smaller).	larger
14	To make the 10 ⁶ larger, shall we add or subtract the change of 2 in the exponent?	We should add 2.
15	Now complete this change of 354.5×10^6 to 3.545×10^9 .	3.545 x 10 ⁸
16	If you were to change 1678.9×10^5 to the proper form of exponential notation, how would you rewrite the decimal numeral part?	Change 1678.9 to 1.6789.
17	In making this change, how many places did you shift the decimal point?	three places.
18	This means we will also have to change the exponent by	3
	We are changing 1678.9×10^5 to $1.6789 \times 10^?$.	
19	Has the decimal numeral part of the expression (which is underlined) been made smaller, or larger?	smaller
20	This means, then, that to keep the same value, the exponent of the 10 will have to be (smaller, larger).	larger
21	Since the decimal point was moved three places, the exponent will be increased by	3
22	Now finish changing 1678.9×10^5 to 1.6789×10^9 by putting in the proper exponent.	1678.9 x 10 ⁵ is changed to
		1.6789 x <u>10⁸</u> .
23	Complete the following: Change 23.45 x 10^4 to 2.345 x 10^7	2.345 x <u>10⁵</u>
24	Change 13579.5 x 10 ⁹ to 1.35795 x 10 [?]	1.35795 x <u>10¹³</u>
25	Change 246.8 x 10^4 to our standard form for exponential notation.	2.468 x 10 ⁶
26	Change 9753.1×10^2 to the standard form of exponential notation.	9.7531 x 10 ⁵

	It may be necessary to go in the other direction in changing the decimal numeral to our standard exponential form. For instance, suppose our expression is	
	$\frac{0.0023}{\text{form.}}$ x 10^5 and we want to put it in our standard	
27	First change the decimal numeral part (which is underlined) into the proper form.	Change 0.0023 to 2.3.
	Now we have 0.0023 x 10 ⁵ changed to 2.3 x 10 [?]	
28	How many places was the decimal point moved in making this change?	Decimal point was moved three places.
29	In changing 0.0023 x 10 ⁵ to 2.3 x 10 ⁷ has the decimal numeral (which is underlined) increased or decreased?	The decimal numeral (which is underlined) increased.
30	To balance or counteract this increase in the size of the decimal numeral the exponent of the power of ten part will have to be(increased, decreased).	decreased.
31	Since the decimal point was shifted 3 places the exponent will be decreased by (how much).	3
	So, the exponent of the ten will be decreased by 3.	
32	Now complete this change: 0.0023 x 10 ⁵ is changed to 2.3 x 10 [?]	0.0023 x 10 ⁵ is changed to
	We always apply the same principle if the decimal numeral part is changed, then the exponent must be changin the opposite way so that the expression continues to represent the same value or quantity.	2.3×10^2 .
33	Try this one: Change 0.00067 x 10^9 to the proper form exponential notation:	6.7 x 10 ⁵
34	Change 0.45 x 10 ² to the proper form in exponential notation or .	4.5 x 10 ¹
	Sometimes it may be necessary to alter our standard form	4.5 x 10
	of exponential notation. Thus, we must be able to chang our decimal numerals or exponents any way we see fit.	çe
	For instance, let's make the following change: Change 8.34×10^5 to $\times 10^7$	
35	What three digits, regardless of the decimal point, will go into the blank space?	are the
36		1 di mi + a
	We may shift the decimal point, but we never change theof the decimal numeral.	digits.
37	Remember, we are changing 8.34×10^5 to $\times 10^7$. What is the change in the exponent?	digits 2
37 38	Remember, we are changing 8.34×10^5 to $\times 10^7$. What is the change in the exponent? Does the exponent increase or decrease?	digits
	Remember, we are changing 8.34×10^5 to $\times 10^7$. What is the change in the exponent?	digits 2 increases decrease

To get this smaller decimal numeral by shifting the 0.0834 decimal point two places we will change 8.34 to 41 $x 10^{7}$ Now, complete this change: 8.34×10^5 changes to 0.0834 If the exponent increases in value, the decimal numeral part must then _____(increase, decrease) in order that decrease the whole expression may not change its value. Change 6.98 x 10^7 to ____ x 10^8 . 0.698 Change 5.67 x 10^{11} to x 10^{10} . 56.7 45 Now let's shift from very large numbers to very small numbers. When we are dealing with very small numbers (in which the exponents are negative) we still apply the same method: An <u>increase</u> in the decimal numeral must be accompanied by a <u>decrease</u> in the exponent. Or, turning this statement around, a decrease in the decimal numeral 46 increase in the exponent. calls for an There is one tricky thing here, however. We are dealing 47 with negative exponents when the number is _____(more, less less) than one. This means we will be comparing negative exponents. 48 Which is greater, -1 or -2? -1 -8 49 Which is smaller, -5 or -8?

If you got both of these last two answers correct and you feel you know how to compare negative numbers, go to Item No. 60.

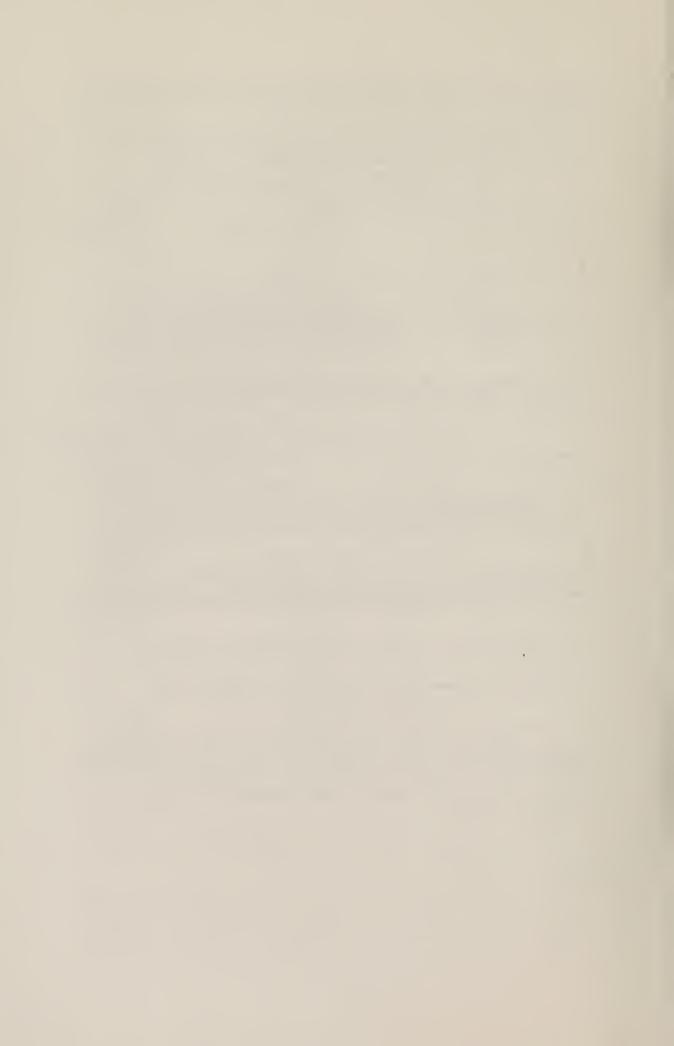
If you missed either one of these last two items, you'll want to continue in this box.

On your paper, draw a rough copy of the diagram of a thermometer similar to the one shown below.

Which reading on the thermometer would indicate a higher (hotter) temperature, A or D? 50 Which would be a higher temperature, a reading at A, +15°, or a reading at G of -15°? A at +15° 51 Which indicates a higher (larger) temperature, C at +5° or Cat +5° E at -5? 52 Which is larger (higher temperature), E at -5 or F at -10? E at -5 53 54 -5 is Looking at your diagram, which is larger, -15 or -5? larger (higher) -5 Which is larger, -5 or -10 ? -15 56 Which is smaller, -15 or -10? -6 57 Which is larger, -8 or -6?

```
58
       Pick out the smallest number: -10, -8, -15, -4
                                                                                     -15
       Pick out the largest number: -2, -20, -200
 59
                                                                                     -2
                                                                                     10-8
60
       Which is larger, 10^{-8} or 10^{-9}?
       Now, let's try the following:
               Change 1.45 x 10^{-5} to x 10^{-6}
61
       By how much has the exponent changed?
                                                                                     It has
                                                                                     changed by 1.
       If the exponent was changed by 1, then the decimal
62
       point will be shifted ____ place.
                                                                                     one
       We are changing 1.45 \times 10^{-5} to
       Which is smaller, 10^{-5} or 10^{-6}?
                                                                                    10-6
63
       Since the new power of ten is smaller, the new decimal numeral must be _____(smaller, larger) to keep the
64
                                                                                     larger
       value the same.
       In changing 1.45 \times 10^{-5} to \times 10^{-6}, we have decided that the decimal point will be shifted one place and that the decimal numeral will be larger.
                                              14.5 x 10<sup>-6</sup>
      What will the new decimal be:
65
                                                               or 0.145 x 10
                                                                                    14.5 x 10<sup>-6</sup>
       Now complete the following:
                                                       x 10-6
                           1.45 \times 10^{-5} =
66
                                                                                     14.5
       See if you can fill in the blank with the proper decimal
       numeral.
                           6.84 \times 10^{-11} = \times 10^{-10}
67
                                                                                    0.684
      In this problem, the value of the exponent increased, therefore the value of the decimal numeral had to
68
                                                                                    decrease
      Try this change: 789 \times 10^{-7} = 7.89 \times 10^{-9}
                                                                                    7.89 \times 10^{-5}
69
      The decimal point was moved ___ places, giving us a smaller decimal numeral, so the exponent had to be increased
70
      Let's try one for a number greater than one, now ---
                             789 \times 10^7 = 7.89 \times 10^?
                                                                                    109
71
        The decimal point was moved two places giving us a
72
                  (smaller, larger) decimal numeral, so the
                                                                                    smaller
      exponent had to be
                                            (increased, decreased) by 2.
                                                                                    increased
```

You may want to practice on the additional problems given in Exercise 3, page 116 of your CHEM Study Laboratory Manual.



MULTIPLICATION AND DIVISION WITH POWERS OF TEN

The multiplication of powers of ten is relatively simple. Since the powers of ten are actually exponents of the base ten, the rules for operations with exponents which you may have developed in a more formal manner in mathematics will apply here.

In multiplying expressions which have the same base (in exponential notation our base is always ten) we need only <u>add the exponents</u> to get the exponent of the product.

For example: $10^3 \times 10^2 = 10^{(3+2)} = 10^5$

Follow this example in doing the following multiplication:

 $10^4 \times 10^2 = 10^{(-+)} =$

10⁵ x 10⁸ equals _____

10 ⁴ x 10 ³ = _____

1

3

5

7 8

10

11

12

 $10^2 \times 10^5 =$

106

1013

10 7

107

The same rule applies even though the exponents may be negative. For example:

$$10^{-2} \times 10^{-3} = 10^{-5}$$

Complete the following:

 $10^{-4} \times 10^{-2} =$

10⁻¹⁰ x 10⁻¹³ =

10⁻¹ x 10⁻² =

Here, when we add negative exponents, the exponent of our result is also

10-6

10-23

10-3

negative

Now suppose we are working with both positive and negative exponents. Again, the same rule of algebraic addition of the exponents will apply.

$$10^5 \times 10^{-2} = 10^3$$

 $10^7 \times 10^{-3} =$

 $10^{-4} \times 10^9 =$

 $10^{-15} \times 10^{20} =$

Here we can see that if the positive exponent has a larger numeral, then the exponent of the result

will also be _____.

104

105

105

nositive

13	It will also be true that if the negative exponent has a larger numeral, then the exponent of the result will also be	negative
	For example: $10^{-15} \times 10^5 = 10^{-10}$	
14	10 ⁻⁸ x 10 ⁶ =	10-2
15	$10^{-7} \times 10^{3} =$	10-4
16	$10^{26} \times 10^{-28} =$	10-2
	In summary of these last four boxes, to add the exponents algebraically when they have different signs we disregard the signs shown, find the difference between the numerals and then put the sign of the larger original exponent in our result. If the exponents have the same sign, just add them. Here are some samples to practice on:	
17	10 ¹² x 10 ⁵ =	1017
18	10 ⁻¹² x 10 ⁵ =	10-7
19	$10^{-12} \times 10^{-5} =$	10-17
20	$10^{12} \times 10^{-5} =$	107
•		
	In multiplying two expressions in exponential notation, the power of ten parts are multiplied together by adding the exponents (as we have already practiced) and the decimal numeral parts are multiplied together to get the decimal numeral part of the answer. For example: 2 x 10 ³ times 3 x 10 ⁴	
	is the same as (2×3) times $(10^3 \times 10^4)$ or 6×10^7	
21	Multiply: 4×10^3 times 2×10^4 which is the same as	
	$(4 \times 2) \text{ times } (10^3 \times 10^4), \text{ or}$	8 x 10 ⁷
	x	6 x 10 ⁹
22	Multiply 3 x 10^2 times 2 x 10^7	12.5 x 10 ⁷
23	Multiply 2.5×10^3 by 5×10^4	15.2 X 10.
	However, this answer, while it is correct, is not in our "standard form" for exponential notation.	
24	Change 12.5 \times 10 ⁷ to the approved form.	1.25 x 10 ⁸
	If you had trouble changing this answer to the "standard" form, refer to the earlier section on "Shifting Decimal Places."	
	_7 _ 2	
25	Multiply 3.5×10^{-7} by 7×10^{-3}	24.5 x 10 ¹⁰
26	Chandra that a training and a contract of	

2.45 x 10⁻⁹

Changing this to our usual form of exponential notation, we get _____.

26

Dividing by powers of ten is just as simple as multiplication. Here the rule of exponents also applies. If the bases are the same (as they always are in powers of ten), the exponent of the divisor (which you are dividing by) is <u>subtracted</u> from the exponent of the dividend (which you are dividing into).

For example:
$$\frac{10^5}{10^3} = 10^{(5-3)} = 10^2$$

27 Using this same form do the following division:

$$\frac{10^8}{10^2} = 10^{(---)} = 10^{---}$$

29 Complete:
$$\frac{10^{27}}{10^{25}} = \frac{10^{25}}{10^{25}}$$

$$\frac{10^8}{10^{12}} = \frac{1}{10^{12}}$$

If you answered these last 3 items correctly, and you understand how to subtract signed numerals skip the next box.

If you missed these last items, or feel you'd like to have a more detailed coverage of the subtraction of signed numerals, go on to the next box.

The algebraic subtraction rule is a simple one. Perhaps you remember from your work in mathematics that you merely reverse the sign of the numeral you are subtracting and go ahead and add.

$$\frac{10^{8}}{10^{2}} = \frac{10^{8}}{10^{(8-2)}} = \frac{10^{6}}{10^{6}} = \frac{10^{7}}{10^{11}}$$

$$\frac{10^{11}}{10^{11}} = \frac{10^{27}}{10^{25}}$$

$$\frac{10^{27}}{10^{(27-25)}} = \frac{10^{2}}{10^{(8-12)}}$$

$$\frac{10^{8}}{10^{12}}$$

$$\frac{10^{12}}{10^{(8-12)}} = \frac{10^{-4}}{10^{(-4-2)}}$$

$$\frac{10^{-4}}{10^{-4}} = \frac{10^{-6}}{10^{-4}}$$

$$\frac{10^{-6}}{10^{-4}} = \frac{10^{-6}}{10^{-4}}$$

$$\frac{10^{-6}}{10^{-2}} = \frac{10^{-6}}{10^{-2}}$$

	For example: From 5 subtract 8	
	Change the sign of 8 from positive to negative, and add algebraically, giving us	
	5 plus -8 = -3	
33	Subtract 6 from 4	(4) + (-6) = -2
34	If we are subtracting a negative number, we merely change its sign from negative to and then add algebraically.	positive
	For example: From 10 subtract -2	
	Here we would change the sign of the (since we are subtracting it) and	- 2
	add algebraically, giving us 10 + 2 =	+12 or 12
	Let's try a new one	
35	Subtract -6 from 10 Here we will change the	
	sign of the(-6, 10)	- 6
36	Our answer will be	(10+6 = 16)
37	From -8 subtract -6	-2 (change
		the -6 to +6 and add)
0.0		
38	Remember, in division of powers of ten, we will (add, subtract) the exponent of the divisor.	subtract
39	Divide $\frac{10^6}{10^4} = \frac{1}{10^4}$	$\frac{10^6}{10^{14}} = 10^2$
40	10-4 =	. ₁₀ -6
	102	(change +2 to -2 and add alge- braically)
41	Try this one: 10 ⁶ =	108
	10-2	(change the sign of the -2 to 2, and add alge-braically)
42	$\frac{10^{-2}}{10^{-5}}$ =	10 ³ (change the sign of the
		-5 to 5 and add alge-braically)
43	$\frac{10^{-9}}{10^{-4}} =$	10 ⁻⁵

Thus far, we have been concerned with only the power of ten part of our exponential notation. As we found in multiplication, the decimal numeral parts of the exponential notation format are divided and this quotient is combined with the power of ten part as shown below:

$$\frac{6.3 \times 10^5}{3 \times 10^2} = \frac{6.3}{3} \times \frac{10^5}{10^2} = 2.1 \times 10^3$$

Try this one, using this same form:

$$\frac{8.6 \times 10^7}{2 \times 10^5} =$$
 x = x

47

50

 $\frac{8.6 \times 10^7}{2 \quad 10^5} = \frac{10^5}{4.3 \times 10^2}$

1.2 x 10⁴

1.05 x 10⁵

Sometimes it will be necessary to put our answer in the standard format of exponential notation.

For instance, if an answer comes out to be 0.018 x 10^7 we will want to change this to 1.8 x $10^{\frac{1}{2}}$.

0.018 x 10⁷ is changed to 1.8 x 10⁵

In all your problems, always put your answer in the standard form for exponential notation.

48 Try:
$$4.59 \times 10^7 \div 9 \times 10^{-4}$$
 equals _____

$$49 \qquad \frac{2.53 \times 10^{-3}}{1.1 \times 10^{3}} = ?$$

$$\frac{182 \times 10^5}{0.13 \times 10^6} = ?$$

5.1 x 10¹⁰
2.3 x 10⁻⁶

1.4 x 10²

You may want to practice on the additional exercise starting at the bottom of the left-hand column of page 117 in your CHEM Study Laboratory Manual.



EXTRACTION OF ROOTS AND RAISING TO A POWER

For the extraction of a root, such as a square root or a cube root, and so forth, of a power of ten, we shall use a simple rule which is based on a more formal development which you may have already gone through in your work in mathematics.

The rule: To extract the root of a power of ten, we merely divide the exponent of the power of ten by the root we wish to extract.

For example, to extract the square root you would divide the exponent of ten by 2. To extract the cube root, you would divide the exponent by 3.

To extract the fourth root, you would divide the exponent of the power by ten by ___.

To find the square root of 10^6 , we divide the exponent by 2. For example,

 $\sqrt{10^6} =$

V10¹⁰ =

1

2

3

6

9

10-8 =

103

105

10-4

To extract the cube root we would merely divide the exponent by the number

 $\sqrt{3/10^6} =$

3/10-12 =

3 10²

10-4

If we have an expression in our exponential notation format, such as 9×10^8 , to take the square root of this whole expression we take the square root of the numerical part, and multiply it by the square root of the power of ten.

For example, $\sqrt{4 \times 10^{10}} = \sqrt{4 \times \sqrt{10^{10}}} = 2 \times 10^5$

Now try these:

 $\sqrt{9 \times 10^{12}} = \sqrt{9} \times \sqrt{10^{12}} =$

 3×10^6 6×10^4

From your work in mathematics you will immediately recognize that we have omitted a very important part of the square root, namely its sign, which should be $\pm 3 \times 10^4$. This positive-negative sign is very important mathematically, but for our purposes the negative roots will not be important. We will be dealing with actual physical quantities in our work in chemistry. Negative weights or negative concentrations, for example, would have no application to our laboratory work. Hence, we can safely use only the positive root and ignore the root.

10 negative Maybe you've noticed that thus far we have carefully selected our examples so that the exponents are nicely, and evenly, divisible. But suppose you have the following problem: $\sqrt{1.6} \times 10^5 = ?$ In this case we could follow our rule and divide the exponent by 2. However, our result, while correct, would leave us with a fractional exponent which is awkward to work with. An easier system is to alter our expression so that it has an exponent which can be evenly divided by our root. In this case, shown above, we would want to change our exponent of ten from 5 to either 6 4 (or any 11 even number) Let's change it to 4 then. $1.6 \times 10^5 = \times 10^4$ 12 If this alteration is not clear to you, review the program on "Shifting the Decimal Place". Now then $\sqrt{1.6 \times 10^5} = \sqrt{16 \times 10^4}$ equals 4×10^{2} If you were given this example, $\sqrt{14.4} \times 10^9$ step would be to change this expression so that the exponent would be ____. even (divisible Let's change the exponent to 8. by 2) $14.4 \times 10^9 = \times 10^8$ 144 15 Now finish this problem by taking the square root and putting your answer into the standard exponential form. $\sqrt{14.4 \times 10^9} = \sqrt{144 \times 10^8} =$ $12 \times 10^4 =$ 1.2 x 105 (positive, negative), but the same method is used. If the number is smaller than one, the exponents will be negati ve 17 3 x 10-6 $\sqrt{9 \times 10^{-12}} =$ 18 If you were given the following square root problem, $\sqrt{16.9 \times 10^{-11}}$, your first step would be to change this expression so that the exponent would be_____ even (or 19 divisible Let's change the exponent to -12: by 2) $16.9 \times 10^{-11} = \times 10^{-12}$ 169 (-12 is smaller than -11, so the decimal numeral must be larger) Now, complete this square root problem:

 $\sqrt{16.9 \times 10^{-11}} = \sqrt{169 \times 10^{-12}} =$

21

 13×10^{-6} , or 1.3×10^{-5}

Perhaps you noticed that all through this program we have selected decimal numerals which are perfect squares and hence the extraction of their square roots were easy. It is assumed that you will be able to extract the square root of any number through the use of logarithms, a slide rule, or "old fashioned arithmetic."

22

23

24

10¹²

As before, if our number is expressed in exponential notation, we operate on the decimal numeral part separately from the powers of ten part. For example:

 $(5 \times 10^3)^2 = 5^2 \times (10^3)^2 = 25 \times 10^6 = 2.5 \times 10^7$ Notice that we changed our answer into the regular exponential notation form.

In the same way, try $(4 \times 10^5)^2 = ($ $)^2 \times ($ $)^2 =$

 $(4)^{2} \times (10^{5})^{2}$ $16 \times 10^{10} = 1.6 \times 10^{11}$

This operation is exactly the same with negative exponents. When a negative exponent is raised to a power, its sign does not change, so it is still _____

negative

```
2.25 x 10<sup>-8</sup>
          (1.50 \times 10^{-4})^2 =
26
          (8.10 \times 10^{-6})^2 =
                                                                                                             6.56 \times 10^{11}
27
          (3.00 \times 10^2)^3 =
                                                                                                              2.70 \times 10^7
28
             4.41 x 10<sup>6</sup>
29
                                                                                                              2.10 x 103
               .225 x 10<sup>5</sup>
30
                                                                                                             3.500 \times 10^{2}
               .6 x 10<sup>-5</sup>
                                                                                                             6.0 x 10-3
31
             62.5 x 10<sup>11</sup>
32
                                                                                                             2.50 \times 10^6
```

You may want to practice on the additional exercises at the bottom of the right-hand column of page 117 of your CHEM Study Laboratory Manual.



ADDITION AND SUBTRACTION OF POWERS OF TEN

1	Suppose you were to perform the following addition:	
	ab + cb = ?	
	The term ab is made up of two factors, a and b. Likewise, the term cb is made up of two factors, namely and	c and b
	So, in the terms ab and cb we have four factors: a, b, c, b.	
2	Which of these factors belongs to ab as well as cb?	b
3	This means we can "factor out" b from the expression ab + cb to give us (a + c)b. This is possible since is a common factor of	
	both ab and cb.	р
1		ı
	Notice that this expression ab + cb is very similar to the problem of adding (or subtracting) quantities in exponential notation, such as	
	$(2.5 \times 10^6) + (1.3 \times 10^6)$	
4	In each of these two expressions, the common factor is	10 ⁶
	Since 10^6 is a common factor in (2.5×10^6) +	
	(1.3 x 10^6) we could rewrite the addition of these two as	6
5	(2.5 + 1.3) x <u>?</u>	10 ⁶
	This tells us that two quantities in exponential notation may be easily added (or subtracted, for that matter) by merely adding (or subtracting) their decimal numeral parts, providing they each have the same exponent or power of ten.	
6	In exponential notation, 2.34 x 10^5 , the power of B C	
	ten part is labeled with the letter	C
7	In 2.34 x 10 ⁵ the decimal numeral part is labeled B C with the letter	В
8	Do these two expressions have the same power of ten?	
	2.34×10^5 5.67×10^5	Yes
	5 5	1
0	Our two expressions are 2.34×10^5 and 5.67×10^5 Since they have the same power of ten, these two	
9	expressions can be added together by merely adding their decimal numeral parts. That means we would add 2.34 to	<u>5.67</u>
	Our problem now looks like this: $(2.34 \times 10^5) + (5.67 \times 10^5) = (2.34 + 5.67) \times 10^5$	
10	When we add the decimal numeral parts together we get x 10 ⁵	8.01
11	Notice that the power of ten part, which was a common factor, is (unchanged, also changed).	unchanged

12	Can these two be added together in their present form? 2.34×10^6 and 3.4×10^7	No (Exponents unequal)
13	Can 4.32×10^{-6} be added to 4.3×10^{-6} ?	Yes (Exponents
14	Can these two be added together in their present form?	are equal)
	3.45×10^8 and 3.45×10^4	No
15	Can 9.2 x 10^7 be added to 9.2 x 10^5 in their present form?	No (Exponents not equal)
16	Could 3.45 x 10^8 be subtracted from 7.4 x 10^8 ?	Yes
17	Can you subtract 3.45×10^8 from 7.4×18^{10} in their present form?	No
	If two exponential expressions have the same exponent or power of ten, then their sum or difference will initially have this same power of ten.	
18	When 3.5 x 10^3 and 2.7 x 10^3 are added together, what will the power of ten be in the answer?	10 ³
	Since the power of ten does not change during the addition or subtraction all we have to be concerned with is the decimal numeral of the exponential notation.	
19	In the expression 2.34×10^7 the decimal numeral part is	2.34
	The decimal numeral parts of the exponential expressions are added or subtracted as though they were by themselves. It is necessary, for instance, to line-up the decimal points before adding or subtracting.	
20	Add: 2.30 x 10 ⁵ + 4.56 x 10 ⁵	6.86 x 10 ⁵
21	Subtract 2.34 x 10 ³ from 5.67 x 10 ³	3.33 x 10 ³
	1	3
22	If the powers of ten or exponents of the two expressions which are being added or subtracted are not the same it is necessary to change the expressions until the tens have the exponent.	same
	Suppose we needed to add these two expressions together: 2.46×10^5 and 3.57×10^6	don't have
23	Can they be added in their present form?	the same powers of ten,
24	Since we can't add them in their present form, we'll have to change one or the other so that the exponents	or exponents) the same
	will be	
	Our two expressions are 2.46 x 10^5 and 3.57 x 10^6 B	
25	To add these, we will want to either change A so that its exponent will be 6, or we will want to change B so that its exponent will be	5 (This will make the exponents equal
		•

Suppose we make each of the exponents equal to 5. 3.57 x 10⁶ Our two expressions are 2.46×10^5 A already has 5 for its exponent so we don't have to do anything to it.
We will have to change B so that its exponent will also be 5. Complete this: B 3.57×10^6 equals $\times 10^5$ 26 (If you missed this last one, you may want to review the program on shifting decimal points.) Note that both B and C have the same value - they are equal - but C now has the same exponent as A (A is 2.46 x 105), and therefore we can add A and C together. 2.46×10^5 (A) $+35.7 \times 10^5$ (C) 38.16 x 10⁵ = The result is ? x ? 27 38.2×10^{5} However, this answer is not in our approved form of exponential notation. 38.2 x 105= Put this answer in its proper form of exponential 28 3.82 x 106 notation. Either of these last two expressions is correct. The second one is in the approved form. 6.47×10^5 29 Try this addition problem, and put your answer in the approved form: (0.68×10^5) 6.8×10^4 plus 5.79×10^5 (Change your powers until they have the same exponent--5, for + (5.79 x 10⁵) $= 6.47 \times 10^{5}$ instance--then add). Subtract 6.80×10^{-11} from 6.88×10^{-10} and put 6.20 x 10⁻¹⁰ 30 (6.88x10⁻¹⁰) your answer in the approved form. - (0.68x10⁻¹⁰) 6.20x10⁻¹⁰ 9.9 x 10⁻⁶ $(1.00 \times 10^{-5}) - (1 \times 10^{-7}) =$ 31 (1.00×10^{-5}) - $(.01 \times 10^{-5})$ $= 0.99 \times 10^{-5} =$ 9.9×10^{-6} $(5.800 \times 10^7) \sim (5.8 \times 10^5)$ 32 5.742 x 10 You may want to practice on the additional problems of Exercises 1 through 6 in the middle of the left-

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hand column of page 117 of your CHEM Study Laboratory

Manual.





